# J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC FINAL EXAM - 03

SOLUTION SET

DURATION - 2 1/2 HR MARKS - 80

## Q-1

01. Differentiate $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ wrt x

$y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$y = x + 2 + \frac{1}{x}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}.$

02. find equation of directrix and the end points of the latus rectum of parabola

$x^2 + 12y = 0$

$x^2 + 12y = 0$

$x^2 = -12y$

On comparing with

$x^2 = -4ay$

$4a = 12$

$a = 3$

equation of directrix : $y = a$

$y = 3$

end points of Latus Rectum

$L \equiv (2a, -a) \equiv (6, -3)$

$L' \equiv (-2a, -a) \equiv (-6, -3)$

03. find the equation of the hyperbola whose vertices are (±2,0) and the foci are (±3,0)

let equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

vertices $\equiv (\pm a, 0) \equiv (\pm 2, 0)$ $\therefore a = 2$

foci $\equiv (\pm ae, 0) \equiv (\pm 3, 0)$ $\therefore ae = 3$

$e = 3/2$

$b^2 = a^2(e^2 - 1)$

$= 4\left(\frac{9}{4} - 1\right)$

$= 5$

04. $\lim_{x \to 0} \left(\frac{7 + 4x}{7 - 5x}\right)^{\frac{1}{x}}$

$= \lim_{x \to 0} \left(\frac{\frac{7 + 4x}{7}}{\frac{7 - 5x}{7}}\right)^{\frac{1}{x}}$

$= \lim_{x \to 0} \left(\frac{1 + \frac{4x}{7}}{1 - \frac{5x}{7}}\right)^{\frac{1}{x}}$

SPLIT THE POWER

$= \lim_{x \to 0} \frac{\left(1 + \frac{4x}{7}\right)^{\frac{1}{x}}}{\left(1 - \frac{5x}{7}\right)^{\frac{1}{x}}}$

SET THE POWER

$= \lim_{x \to 0} \frac{\left\{\left(1 + \frac{4x}{7}\right)^{\frac{1}{\frac{4x}{7}}}\right\}^{\frac{4}{7}}}{\left\{\left(1 - \frac{5x}{7}\right)^{\frac{1}{\frac{-5x}{7}}}\right\}^{\frac{-5}{7}}}$

$= \frac{e^{\frac{4}{7}}}{e^{\frac{-5}{7}}} = e^{\frac{4}{7} + \frac{5}{7}} = e^{\frac{9}{7}}$

05. $\lim_{x \to 0} \dfrac{\log(2+x) - \log 2}{x}$

$$= \lim_{x \to 0} \frac{\log\left[\frac{2+x}{2}\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log\left[1 + \frac{x}{2}\right]}{x}$$

$$= \lim_{x \to 0} \frac{\frac{1}{2}\log\left[1 + \frac{x}{2}\right]}{\frac{x}{2}}$$

$$= \frac{1}{2}(1) = \frac{1}{2}$$

06. find $\dfrac{dy}{dx}$ if $y = \sqrt{x} \, . \cot x$

$y = \sqrt{x} \, . \cot x$

$$\frac{dy}{dx} = \sqrt{x} \frac{d}{dx} \cot x + \cot x \frac{d}{dx} \sqrt{x}$$

$$= \sqrt{x}(-\operatorname{cosec}^2 x) + \cot x \frac{1}{2\sqrt{x}}$$

$$= -\sqrt{x} . \operatorname{cosec}^2 x + \frac{\cot x}{2\sqrt{x}}$$

07. $\lim_{x \to 3} (x-2)^{\frac{1}{(x-3)}}$

$x = 3 + h$

$$= \lim_{h \to 0} (3 + h - 2)^{\frac{1}{3+h-3}}$$

$$= \lim_{h \to 0} (1 + h)^{\frac{1}{h}}$$

$$= e$$

*08.* **Prove** : $\sin^2\left[\frac{\pi}{8}\right] + \sin^2\left[\frac{3\pi}{8}\right] = 1$

$$\sin^2\left[\frac{\pi}{8}\right] + \sin^2\left[\frac{3\pi}{8}\right]$$

$$= \sin^2\left[\frac{\pi}{8}\right] + \sin^2\left[\frac{4\pi}{8} - \frac{\pi}{8}\right]$$

$$= \sin^2\left[\frac{\pi}{8}\right] + \sin^2\left[\frac{\pi}{2} - \frac{\pi}{8}\right]$$

$$= \sin^2\left[\frac{\pi}{8}\right] + \cos^2\left[\frac{\pi}{8}\right]$$

$$= 1$$

**Q2. (A)**

# Q - 2A

**01. Prove :**

$$\frac{\sin 5A.\sin 11A - \sin 7A \sin 9A}{\sin 5A.\cos 11A - \sin 7A.\cos 9A} = \tan 4A$$

$$= \frac{2\sin 11A.\sin 5A - 2\sin 9A \sin 7A}{2\cos 11A.\sin 5A - 2\cos 9A \sin 7A}$$

$$= \frac{\cos 6A - \cos 16A - \cos 2A - \cos 16A}{\sin 16A - \sin 6A - \sin 16A - \sin 2A}$$

$$= \frac{\cos 6A - \cos 16A - \cos 2A + \cos 16A}{\sin 16A - \sin 6A - \sin 16A + \sin 2A}$$

$$= \frac{\cos 6A - \cos 2A}{\sin 2A - \sin 6A}$$

$$= \frac{-2\sin\left(\frac{6A+2A}{2}\right)\sin\left(\frac{6A-2A}{2}\right)}{2\cos\left(\frac{2A+6A}{2}\right)\sin\left(\frac{2A-6A}{2}\right)}$$

$$= -\frac{2\sin 4A . \sin 2A}{2\cos 4A. \sin(-2A)}$$

$$= \frac{2\sin 4A . \sin 2A}{2\cos 4A. \sin 2A}$$

$$= \tan 4A$$

**02.** **Prove** : $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x$

$\cos^{-1}(4x^3 - 3x)$

Put $x = \cos\theta$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta$

$= 3\cos^{-1}x$

**03.** **Prove :**

$$\sin A \cdot \tan \frac{A}{2} + 2\cos A = \frac{2}{1 + \tan^2 \frac{A}{2}}$$

LHS

$$= \sin A \cdot \tan \frac{A}{2} + 2\cos A$$

$$= 2\sin\frac{A}{2} \cdot \cos\frac{A}{2} \cdot \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} + 2\cos^2\frac{A}{2} - \sin^2\frac{A}{2}$$

$$= 2\sin^2\frac{A}{2} + 2\cos^2\frac{A}{2} - 2\sin^2\frac{A}{2}$$

$$= 2\cos^2\frac{A}{2}$$

RHS

$$= \frac{2}{1 + \tan^2 \frac{A}{2}}$$

$$= \frac{2}{1 + \frac{\sin^2(A/2)}{\cos^2(A/2)}}$$

$$= \frac{2}{\frac{\cos^2(A/2) + \sin^2(A/2)}{\cos^2(A/2)}}$$

$$= 2\cos^2(A/2)$$

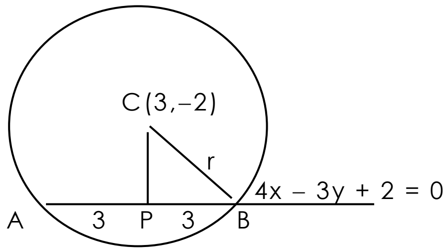
LHS = RHS

# Q - 2B

**Q2. (B)**

01.

Find equation of circle with center (3 ,−2) and which cuts off a chord of length 6 on line $4x - 3y + 2 = 0$




STEP 1

$$CP = \left|\frac{4(3) - 3(-2) + 2}{\sqrt{4^2 + 3^2}}\right|$$

$$= \left|\frac{12 + 6 + 2}{5}\right|$$

$$= 4$$

STEP 2

$r^2 = CP^2 + PB^2$

$= 4^2 + 3^2$

$= 25$

$r = 5$

STEP 3

$C(3,-2)$ , $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y + 2)^2 = 5^2$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 25$

$x^2 + y^2 - 6x + 4y + 13 - 25 = 0$

$x^2 + y^2 - 6x + 4y - 12 = 0$

**02.**

Find the eccentricity , co-ordinates of foci , Equation of directrices and the length of latus rectum of the ellipse $2x^2 + 5y^2 = 10$

$$\frac{2x^2}{10} + \frac{5y^2}{10} = 1$$

$$\frac{x^2}{5} + \frac{y^2}{2} = 1$$

$a^2 = 5 \quad \therefore\ a = \sqrt{5}$

$b^2 = 2 \quad \therefore\ b = \sqrt{2} \qquad a > b$

Eccentricity

$b^2 = a^2(1 - e^2)$

$2 = 5(1 - e^2)$

$\frac{2}{5} = 1 - e^2$

$e^2 = 1 - \frac{2}{5}$

$e^2 = \frac{3}{5}$

✓ $e = {}^{\sqrt{3}}/_{\sqrt{5}}$

$ae = \sqrt{5} \times {}^{\sqrt{3}}/_{\sqrt{5}} = \sqrt{3}$

$\frac{a}{e} = \frac{\sqrt{5}}{{}^{\sqrt{3}}/_{\sqrt{5}}} = \frac{5}{\sqrt{3}}$

✓ foci $\equiv (\pm ae, 0) \equiv (\pm\sqrt{3}, 0)$

✓ eq. of directrices : $x = \pm\, {}^{a}/_{e}$

$x = \pm\, {}^{5}/_{\sqrt{3}}$

✓ length of major axis $= 2a = 2\sqrt{5}$

✓ length of minor axis $= 2b = 2\sqrt{2}$

✓ length of latus rectum $= \frac{2b^2}{a} = \frac{4}{\sqrt{5}}$

**03.**

Find the inverse and the range of the function : $g(x) = 3 - 5x\ ;\ -1 \le x \le 3$

range of the function

$g(x) = 3 - 5x\ ;\ -1 \le x \le 3$

$-1 \le x \le 3$

$-5 \le 5x \le 15$

$5 \ge -5x \ge -15$

$5 + 3 \ge 3-5x \ge -15 + 3$

$8 \ge g(x) \ge -12$

Range of g(x) : $[-12,8]$

Inverse of function

$g(x) = 3 - 5x$

$y = 3 - 5x$

$5x = 3 - y$

$x = \frac{1}{5}(3 - y)$

$g^{-1}(x) = \frac{1}{5}(3 - x)$

**Q3(A)**

# Q - 3A

**01.** if $f(x) = \frac{x-4}{4x-1}$; then show that $fof(x) = x$

$$f(x) = \frac{x-4}{4x-1}$$

$$f(f(x)) = \frac{f(x)-4}{4f(x)-1}$$

$$= \frac{\frac{x-4}{4x-1} - 4}{4\left(\frac{x-4}{4x-1}\right) - 1}$$

$$= \frac{\frac{x-4-16x+4}{4x-1}}{\frac{4x-16-4x+1}{4x-1}}$$

$$= \frac{-15x}{-15}$$

$= x$ ………. proved

**02.** Solve the following equations using Cramer's Rule

$x + y = 3$ , $y + z = 5$ , $x + z = 4$

$$D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-1) = 1 + 1 = 2$$

$$D_x = \begin{vmatrix} 3 & 1 & 0 \\ 5 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 3(1-0) - 1(5-4) = 3 - 1 = 2$$

$$D_y = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1(5-4) - 3(0-1) = 1 + 3 = 4$$

$$D_z = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 1 & 0 & 4 \end{vmatrix} = 1(4-0) - 1(0-5) + 3(0-1) = 4 + 5 - 3 = 6$$

$x = \frac{D_x}{D} = 1$ ; $y = \frac{D_y}{D} = 2$ ; $z = \frac{D_z}{D} = 3$

SS {1 , 2 , 3}

**03.** $y = \frac{x^3 - \sin x}{\cos x}$ . Find dy/dx

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(x^3 - \sin x) - (x^3 - \sin x)\frac{d\cos x}{dx}}{\cos^2 x}$$

$$= \frac{\cos x\,(3x^2 - \cos x) - (x^3 - \sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x\,(3x^2 - \cos x) + (x^3 - \sin x)\sin x}{\cos^2 x}$$

$$= \frac{3x^2\cos x - \cos^2 x + x^3\sin x - \sin^2 x}{\cos^2 x}$$

$$= \frac{3x^2\cos x + x^3\sin x - (\cos^2 x + \sin^2 x)}{\cos^2 x}$$

$$= \frac{3x^2\cos x + x^3\sin x - 1}{\cos^2 x}$$

Q3 (B)

# Q - 3B

**01.**

$$\lim_{x \to 0} \frac{2 \sin x^{o} - \sin 2x^{o}}{x^3}$$

$$= \lim_{x \to 0} \frac{2 \sin x - 2 \sin x \cdot \cos x}{x^3}$$

$$= \lim_{x \to 0} \frac{2 \sin x \ (1 - \cos x)}{x^3}$$

$$= \lim_{x \to 0} 2 \ \frac{\sin x \cdot 2 \sin^2 (x/2)}{x^3}$$

$$= \lim_{x \to 0} 4 \ \frac{\sin x^{o}}{x} \cdot \frac{\sin^2 (x/2)^{o}}{x^2}$$ DISTRIBUTE

SQUARE–SQUARE THE WHOLE SQUARE

$$= \lim_{x \to 0} 4 \ \frac{\sin x}{x} \left( \frac{\sin (x/2)}{x} \right)^2$$

**NOTE : ANGLES ARE IN DEGREES , NEED TO CONVERT TO RADIAN**

$$= \lim_{x \to 0} 4 \ \frac{\sin \frac{\pi x}{180}}{x} \left( \frac{\sin \frac{\pi x}{360}}{x} \right)^2$$

$$= \lim_{x \to 0} 4 \frac{\pi}{180} \ \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \left( \frac{\pi}{360} \frac{\sin \frac{\pi x}{360}}{\frac{\pi x}{360}} \right)^2$$

$$= 4 \cdot \frac{\pi}{180} \left( \frac{\pi}{360} \right)^2$$

$$= 4 \cdot \frac{\pi}{180} \left( \frac{\pi}{2 \times 180} \right)^2$$

$$= \left( \frac{\pi}{180} \right)^3$$

**02.**

the demand function is given as

$P = 175 + 9D + 25D^2$

Find the total revenue , average revenue and marginal revenue when demand is 10

SOLUTION

Total Revenue

$R = pD$

$= (175 + 9D + 25D^2).D$

$= 175D + 9D^2 + 25D^3$

Put D = 10

$= 1750 + 900 + 25000$

$= 27650$

NOTE

(average revenue is the revenue obtained by selling one item and that is the price of the item (p) )

Average Revenue

$= P$

$= 175 + 9D + 25D^2$

Put D = 10

$= 175 + 90 + 2500$

$= 2765$

Marginal Revenue when D = 10

$= \frac{dR}{dD}$

$= 175 + 18D + 75D^2$

Put D = 10

$= 175 + 180 + 7500$

$= 7855$

**03.** $y = \dfrac{\sin\sqrt{x^2+2} + \log(xe^x)}{5^{x\tan x}}$

HINTS

$$\frac{d}{dx}\sin\sqrt{x^2+2} = \cos\sqrt{x^2+2}\,\frac{d}{dx}\sqrt{x^2+2} = \frac{x\cos\sqrt{x^2+2}}{\sqrt{x^2+2}}$$

$$\frac{d}{dx}\log(xe^x) = \frac{1}{xe^x}\frac{d}{dx}(xe^x) = \frac{e^x(x+1)}{xe^x} = \frac{x+1}{x}$$

$$\frac{d}{dx}\left[\sin\sqrt{x^2+2} + \log(xe^x)\right] = \frac{x\,.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \frac{x+1}{x}$$

$$\frac{d}{dx}5^{x\tan x} = 5^{x\tan x}.\log 5\,.\frac{d}{dx}x\tan x = 5^{x\tan x}.\log 5\,.(x\sec^2 x + \tan x)$$

now

$$\frac{dy}{dx} = \frac{5^{x\tan x}\dfrac{d}{dx}\left[\sin\sqrt{x^2+2} + \log(xe^x)\right] - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]\dfrac{d}{dx}5^{x\tan x}}{(5^{x\tan x})^2}$$

$$= \frac{5^{x\tan x}\left(\dfrac{x\,.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \dfrac{x+1}{x}\right) - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]5^{x\tan x}.\log 5\,.(x\sec^2 x + \tan x)}{(5^{x\tan x})^2}$$

$$= \frac{5^{x\tan x}\left(\dfrac{x\,.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \dfrac{x+1}{x} - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]\log 5\,.(x\sec^2 x + \tan x)\right)}{(5^{x\tan x})^2}$$

$$= \frac{\left(\dfrac{x\,.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \dfrac{x+1}{x} - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]\log 5\,.(x\sec^2 x + \tan x)\right)}{5^{x\tan x}}$$

# SECTION - II

# Q - 4

**Q4**

**01.**

Check the consistency of the following data
(AB) = 200 , N = 1000 , (A) = 150 , (B) = 300

| | B | β | TOTAL |
|---|---|---|---|
| A | **(AB) = 200** | **(Aβ) = – 50** | **(A) = 150** |
| α | **(αB) = 100** | **(αβ) = 750** | **(α) = 850** |
| | **(B) = 300** | **(β) = 700** | **N =1000** |

Since (Aβ) < 0 , the data is INCONSISTENT

**02.**

how many four digits numbers greater than 4000 can be formed using the digits
2 , 3, 4 , 5 , 6 , 7
if no repetitions of digits are allowed

the thousands place can be filled by any of the digits 4 , 5, ,6 & 7 in $^4P_1$ ways

having done that ,

the remaining 3 places can be filled by any of the remaining 5 digits in $^5P_3$ ways

By Fundamental principle of Multiplication

Total numbers formed = $^4P_1 \times {}^5P_3$
= 4 x 60
= 240

**03.**

Find the missing frequencies
(αB) = 500 , (B) = 600 , (α) = 800 , (β) = 1000

| | B | β | TOTAL |
|---|---|---|---|
| A | **(AB) = 100** | **(Aβ) = 700** | **(A) = 800** |
| α | **(αB) = 500** | **(αβ) = 300** | **(α) = 800** |
| | **(B) = 600** | **(β) = 1000** | **N =1600** |

**04.**

find the no of sides of a polygon which has 54 diagonals

let the number of sides = n ∴ n points

2 points define a line

∴ no of lines that can be drawn are $^nC_2$ out of which n are sides

∴ no of diagonals = $^nC_2 - n$

$$^nC_2 - n = 54$$

$$\frac{n(n-1)}{2} - n = 54$$

$$n\left(\frac{n-1}{2} - 1\right) = 54$$

$$n(n-3) = 108$$

$$n(n-3) = 12.\ 9$$

On Comparing , n = 12

**05.**

For the following data calculate the Price Index Number using Simple Aggregate method

| Commodity | P | Q | R | S | T |
|---|---|---|---|---|---|
| $p_o$(1995) | 10 | 25 | 14 | 20 | 30 |
| $p_1$(2000) | 32 | 40 | 20 | 45 | 70 |

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100 = \frac{207}{99} \times 100 = 209.1$$

**06.** Find Cost of Living index number

| | Food | Rent | Clothing | Fuel &Light | Misc |
|---|---|---|---|---|---|
| I | 410 | 150 | 343 | 248 | 285 |
| W | 45 | 15 | 12 | 8 | 20 |

| I | W | IW |
|---|---|---|
| 410 | 45 | 18450 |
| 150 | 15 | 2250 |
| 343 | 12 | 4116 |
| 248 | 8 | 1984 |
| 285 | 20 | 5700 |

ΣIW = 32500

$$CLI = \frac{\Sigma IW}{\Sigma W} = \frac{32500}{100} = 325$$

**07.**
two unbiased dice are rolled . Find the probability that the sum of numbers on the upper most faces is a perfect square or a number less than 5

let

A ≡ sum of numbers is a perfect square (i.e. 4 , 9 )
≡ { (1,3) , (2,2) , (3,1) , (3,6) , (4,5) , (5,4) (6,3) }

$P(A) = {}^{7}/_{36}$

B ≡ sum of numbers is less than 5 (i.e. 2 , 3 ,4 )
≡ { (1,1) , (1,2) , (2,1) , (1,3) , (2,2) , (3,1) }

$P(B) = {}^{6}/_{36}$

A∩B = {(1,3) , (2,2) , (3,1) }

$P(A \cap B) = {}^{3}/_{36}$

E ≡ sum of numbers on the upper most faces is a perfect square or a number less than 5

$E \equiv A \cup B$

$P(E) \equiv P(A \cup B)$

$P(E) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{36} + \frac{6}{36} - \frac{3}{36} = \frac{10}{36} = \frac{5}{18}$

**08.**
Compute 5 yearly moving average values

Year : 1989 1990 1991 1992 1993 1994
Profit: 53 79 76 66 69 94

Year : 1995 1996
Profit : 88 98

| Year | Profit | 5 year MOVING TOTAL T | 5 year MOVING AVG $^{T}/_{5}$ |
|---|---|---|---|
| 1989 | 53 | | |
| 1990 | 79 | | |
| 1991 | 76 | 53+79+76+66+69=343 | 343/5 = 68.6 |
| 1992 | 66 | 79+76+66+69+94=384 | 384/5 = 76.8 |
| 1993 | 69 | 76+66+69+94+88=393 | 393/5 = 78.6 |
| 1994 | 94 | 66+69+94+88+98=415 | 415/5 = 83 |
| 1995 | 88 | | |
| 1996 | 98 | | |

**Q5(A)**

# Q - 5A

**01.** If A and B are independent events such that P(A) = 2/5 and P(B) = 1/3 , find **(i)** $P(A \cap B)$ **(ii)** $P(A \cup B)$ **(iii)** $P(A' \cap B')$

$P(A \cap B) = P(A) \times P(B) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{2}{5} + \frac{1}{3} - \frac{2}{15}$

$= \frac{6 + 5 - 2}{15} = \frac{3}{5}$

$P(A' \cap B') = P(A') \times P(B') = \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$

**02.**

10 identical components four of which are defective , 2 components are drawn in succession without replacement Find the probability of

a) both the components are defective

b) atleast one of them is non defective

a) E ≡ both the components are defective
E ≡ A∩B

$P(E) = P(A \cap B)$

$= P(A) \times P(B/A)$

$= \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

b) E ≡ atleast one of the component is non defective
E' ≡ none of the components is non defective
(i.e both the components are defective )

P(E') = 2/15 ……. From (a)

P(E) = 1−P(E')

= 13/15

**03.** Obtain the trend value using 4 – yearly moving average

| Year | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|
| IMR | 114 | 97 | 80 | 74 | 68 | 58 | 49 |

| Year | IMR | 4 Year Moving total | 4 year moving total Centered (T) | 4 year moving avg Centered (T/8) |
|---|---|---|---|---|
| 2004 | 114 | | | |
| 2005 | 97 | | | |
| | | 114+97+80+74=365 | | |
| 2006 | 80 | | 365+319 = 684 | 684/8 = 85.5 |
| | | 97+80+74+68 =319 | | |
| 2007 | 74 | | 319+280 = 599 | 599/8 = 74.875 |
| | | 80+74+68+58 =280 | | |
| 2008 | 68 | | 280+249 = 529 | 529/8 = 66.125 |
| | | 74+68+58+49 =249 | | |
| 2009 | 58 | | | |
| 2010 | 49 | | | |

**Q5.(B)**

# Q - 5B

**01.**

| $p_0$ | $q_0$ | $p_1$ | $q_1$ | $p_1q_0$ | $p_1q_1$ | $p_0q_0$ | $p_0q_1$ |
|---|---|---|---|---|---|---|---|
| 22 | 10 | 25 | 30 | 250 | 750 | 220 | 660 |
| 34 | 12 | 35 | 40 | 420 | 1400 | 408 | 1360 |
| 28 | 15 | 25 | 25 | 375 | 625 | 420 | 700 |
| 26 | 14 | 25 | 10 | 350 | 250 | 364 | 260 |
| 30 | 11 | 35 | 10 | 385 | 350 | 330 | 300 |
| | | | | 1780 | 3375 | 1742 | 3280 |
| | | | | $\Sigma p_1q_0$ | $\Sigma p_1q_1$ | $\Sigma p_0q_0$ | $\Sigma p_0q_1$ |

$$P_{01}(L) = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100 = \frac{1780}{1742} \times 100 = 102.2$$

**LOG CALC**

**3.2504**
**−3.2410**
**AL(0.0094)**
**1.022**

$$P_{01}(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \frac{3375}{3280} \times 100 = 102.9$$

**LOG CALC**

**3.5282**
**−3.5159**
**AL(0.0123)**
**1.029**

$$P_{01}(DB) = \frac{P01(L) + P01(P)}{2} = \frac{102.2 + 102.9}{2} = 102.55$$

**02.** How many 5 digit numbers can be formed with digits 0 , 2 , 3 , 4 , 6 , 7 , 8 , 9 if repetitions of the digit is not allowed . How many of these are odd . How many of these are even

**Step 1 : 5 – digit numbers formed**

Ten Thousand place can be filled by any one of the 7 digits ('0' excluded) in 7 ways

Having done that , remaining 4 places can be filled by any of remaining 7 digits in ${}^{7}P_{4}$ ways

By fundamental principle of Multipliation ,

Numbers formed = 7 x ${}^{7}P_{4}$ = 7 x 840 = 5880

**Step 2 : Odd Numbers**

Unit place can be filled by any one of digits 3 , 7 , 9 in ${}^{3}P_{1}$ ways

Having done that ,

Ten Thousand place can be filled by any one the remaining 6 digits ('0' excluded & unit place already filled ) in ${}^{6}P_{1}$ ways

Having done that , remaining 3 places can be filled by any of remaining 6 digits in ${}^{6}P_{3}$ ways

By fundamental principle of Multipliation ,

No. of Odd Numbers formed = ${}^{3}P_{1}$ x ${}^{6}P_{1}$ x ${}^{6}P_{3}$ = 3 x 6 x 120 = 2160

**Step 3 : Even Numbers**

No. of Even numbers formed = 5880 – 2160 = 3720

**03.**

| | Typhoid | No typhoid |
|---|---|---|
| Chloromycin | 35 | 850 |
| No Chloromycin | 365 | 3250 |

Is Chloromycin effective in preventing typhoid

NOTE : We will find yules coefficient of association between chloromycin and no typhoid to examine its effectiveness in preventing typhoid and hence

A : person is administered with 'CHLOROMYCIN'
B : person is suffereing from 'NO TYPHOID'

| | No Typhoid B | Typhoid β |
|---|---|---|
| Chloromycin A | (AB) = 850 | (Aβ) = 35 |
| No Chloromycin α | (αB)=3250 | (αβ)= 365 |

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$= \frac{850x365 - 35 \times 3250}{850x365 + 35 \times 3250}$$

$$= \frac{310250 - 113750}{310250 + 113750}$$

$$= \frac{196500}{424000}$$

$$= \frac{1965}{4240}$$

$$= 0.4634$$

LOG CALC.
3. 2934
– 3. 6274
AL $\bar{1}$. 6660
0. 4634

COMMENT

There is significant amount of positive asscociation between A and B and hence Chloromycin is effective in preventing Typhoid

**Q6. (A)**

# Q - 6A

**01.**

Compute the standard deviation for the following data

| marks more than | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| no of students | 50 | 46 | 40 | 20 | 10 | 3 |

| CI | f | x | $u = \frac{x-25}{10}$ | fu | $fu^2$ |
|---|---|---|---|---|---|
| 0 – 10 | 4 | 5 | –2 | – 8 | 16 |
| 10 – 20 | 6 | 15 | –1 | – 6 | 6 |
| 20 – 30 | 20 | 25 | 0 | 0 | 0 |
| 30 – 40 | 10 | 35 | 1 | 10 | 10 |
| 40 – 50 | 7 | 45 | 2 | 14 | 28 |
| 50 – 60 | 3 | 55 | 3 | 9 | 27 |
| | 50 | | | 19 | 87 |

$$\sigma u = \sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{87}{50} - \left(\frac{19}{50}\right)^2}$$

$$= \sqrt{\frac{4350 - 361}{50^2}}$$

$$= \sqrt{\frac{3989}{2500}}$$

$$= 1.263$$

$$\sigma x = 10 \times \sigma u = 12.63$$

LOG CALC

3.6009
– 3.3979
0.2030
2
AL(0.1015)
1.263

**02.** a boy has 3 library tickets and 6 books of his interest in the library . Of these 6 books , he does not want to borrow Math II , unless Math I is borrowed . In how many ways can he choose three books to be borrowed .

Case 1 : <u>Math II is borrowed</u>

Since Math II is borrowed , the boy must have borrowed Math I

He has to now select remaining 1 book from the remaining 4 books .

This can be done in ${}^4C_2$ = 6 ways

Case 2 : <u>Math II is NOT borrowed</u>

Since Math II is NOT borrowed , the boy has to now select 3 books from the remaining 5 books .

This can be done in ${}^5C_3$ = 10 ways

<u>By Fundamental principle of Addition</u> : Total ways = 10 + 6 = 16

**03.** $^{2n}C_3 : {}^{n}C_2 = 52 : 3$

$$\frac{^{2n}C_3}{^{n}C_2} = \frac{52}{3}$$

$$\frac{\frac{2n!}{(2n-3)!.3!}}{\frac{n!}{(n-2)!2!}} = \frac{52}{3}$$

$$\frac{2n!}{(2n-3)!.3!} \cdot \frac{(n-2)!2!}{n!} = \frac{52}{3}$$

$$\frac{2n(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \cdot \frac{(n-2)!}{n(n-1)(n-2)!} \cdot \frac{2!}{3!} = \frac{52}{3}$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)} \cdot \frac{1}{3} = \frac{52}{3}$$

$$\frac{2n(2n-1)2(n-1)}{n(n-1)} = 52$$

$$4(2n-1) = 52$$

$$8n - 4 = 52$$

$$8n = 56$$

$$n = 7$$

## Q - 6B

**Q6. (B)**

**01.** two fair dice are thrown . Find the probability that sum of points is 9 given that its exceeds 8

A ≡ sum of points is 9
≡ { (3,6) , (4,5) , (5,4) , (6,3) }

$P(A) = 4/36$

B ≡ sum of points exceeds 8
≡ { (3,6) , (4,5) , (5,4) , (6,3) , (4,6) , (5,5) , (6,4) , (5,6) , (6,5) , (6,6)}

$P(B) = 10/36$

$A \cap B$ ≡ { (3,6) , (4,5) , (5,4) , (6,3) }

$P(A \cap B) = 4/36$

E ≡ sum of points is 9 given that its exceeds 8

E ≡ A | B

$$P(E) = P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{4/36}{10/36} = \frac{2}{5}$$

**02.**

Fit a trend line by least square method for the following data which represents production in thousand units of a small scale industry

| Year | 1980 | 1981 | 1982 | 1983 | 1984 |
|---|---|---|---|---|---|
| Prod$^n$ | 12 | 15 | 18 | 17 | 16 |

| t | y | u = t − 1982 | $u^2$ | yu |
|---|---|---|---|---|
| 1980 | 12 | −2 | 4 | −24 |
| 1981 | 15 | −1 | 1 | −15 |
| 1982 | 18 | 0 | 0 | 0 |
| 1983 | 17 | 1 | 1 | 17 |
| 1984 | 16 | 2 | 4 | 32 |
| | 78 | 0 | 10 | 10 |

trend line :

$y = a + bu \longrightarrow yu = au + bu^2$

$\Sigma y = na + b\Sigma u \qquad \Sigma yu = a\Sigma u + b\Sigma u^2$

$78 = 5a \qquad 10 = b(10)$

$a = 15.6 \qquad b = 1$

$$y = 15.6 + u \; ; \quad u = t - 1982$$

**03.**

| $p_0$ | $q_0$ | $p_1$ | $q_1$ | $p_1q_0$ | $p_1q_1$ | $p_0q_0$ | $p_0q_1$ |
|---|---|---|---|---|---|---|---|
| 10 | 12 | 40 | 3 | 480 | 120 | 120 | 30 |
| 20 | 2 | 25 | 8 | 50 | 200 | 40 | 160 |
| 30 | 3 | 50 | 27 | 150 | 1350 | 90 | 810 |
| 60 | 9 | 90 | 36 | 810 | 3240 | 540 | 2160 |
| | | | | 1490 | 4910 | 790 | 3160 |
| | | | | $\Sigma p_1q_0$ | $\Sigma p_1q_1$ | $\Sigma p_0q_0$ | $\Sigma p_0q_1$ |

$$P_{01}(L) = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

$$= \frac{1490}{790} \times 100$$

$$= 188.7$$

LOG CALC

3.1732
−2.8976
AL(0.2756)
1.887

$$P_{01}(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{4910}{3160} \times 100$$

$$= 155.3$$

LOG CALC

3.6911
−3.4997
AL(0.1914)
1.553